Neutrinoless double beta decay with and without Majoron-like boson emission in a 3-3-1 model

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We consider the contributions to the neutrinoless double beta decays in a $SU(3)_L \otimes U(1)_N$ electroweak model. We show that for a range of the parameters in the model there are diagrams involving vector-vector-scalar and trilinear scalar couplings which can be potentially as contributing as the light massive Majorana neutrino exchange one. We use these contributions to obtain constraints upon some mass scales of the model, like the masses of the new charged vector and scalar bosons. We also consider briefly the decay in which besides the two electrons a Majoron-like boson is emitted.

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I. INTRODUCTION

The issue of neutrino masses continues to be a golden plate in elementary particle physics. Although data coming from solar [1], atmospheric [2], and the accelerator LSND [3] neutrino experiments strongly suggest that neutrinos must be massive particles, direct measurements did not obtain any positive result [4].

It is a very well known fact that if neutrinos are massive Majorana particles it should exist the neutrinoless double beta $(\beta\beta)_{0\nu}$ decay [5,6]. If the neutrino mass is the main effect that triggers this decay, the decay lifetime is proportional to (for the case of light neutrinos)

$$\langle M_\nu \rangle = \sum_i U_{ei}^2 m_{\nu_i}, \qquad (1)$$

where U_{ei}; $i = 1, 2, 3$ denote the elements of a mixing matrix that relates symmetry ν_α; $\alpha = e, \mu, \tau$ and mass eigenstates ν_i through the relation $\nu_\alpha = \sum_i U_{\alpha i} \nu_i$; and m_{ν_i} are the neutrino masses. Experimentally a half life limit $T_{1/2}^{0\nu} > 1.8 \times 10^{25}$ yr implies [7]

$$\langle M_\nu \rangle < 0.2 \text{ eV}. \qquad (2)$$

The important point is that the $(\beta\beta)_{0\nu}$ decay probes the physics beyond the standard model. In particular the observation of this decay would be an evidence for a massive Majorana neutrino although it could say nothing about the value of the mass. This is because although right-handed currents and/or scalar-bosons may affect the decay rate, it has been shown that whatever the mechanism of this decay is, a nonvanishing neutrino mass is required for the decay to take place [8]. However, this does not mean that the neutrino mass is necessarily the main factor triggering this decay. In some models the $(\beta\beta)_{0\nu}$ decay can proceed with arbitrary small neutrino mass via scalar boson exchange [9]. The mechanism involving a trilinear interaction of the scalar bosons was proposed in Ref. [10] in the context of a model with $SU(2) \otimes U(1)$ symmetry with doublets and a triplet of scalar bosons. However, since in this type of models there is no large mass scale, it was shown in Refs. [11] that the contribution of the trilinear interactions are in fact negligible. In general, in models with that symmetry, a fine tuning is needed if we want that the trilinear terms give important contributions to the $(\beta\beta)_{0\nu}$ decay [8,12].

Here we will show that in a model with gauge symmetry $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$ (3-3-1 model by short) [13], which has a rich Higgs bosons sector as in the multi-Higgs extensions of the standard model, there are new contributions to the $(\beta\beta)_{0\nu}$ decay. However, unlike the latest sort of models, a fine tuning of the parameters of the 3-3-1 model is not necessary since some trilinear couplings, which have mass dimension, could imply an enhancement of the respective amplitudes (See. Sec. III).

We will use the following strategy: First, we consider the several new contributions to the $(\beta\beta)_{0\nu}$ decay introduced by the 3-3-1 model. Next, once this decay has not experimentally been seen, we will consider the usual standard model amplitude (that would arise with massive Majorana neutrinos) as the reference one and make the assumption that all the new amplitudes are at most as contributing as this one. Hence, we can obtain constraints on some typical mass scale 3-3-1 parameters. The new contributions to the $(\beta\beta)_{0\nu}$ decay are of the short

range type [14]. Since the respective matrix elements are different from those of the long range contributions (the exchange of a light-Majorana neutrino) our results should be considered only as an indication of the possible large contributions to this decay in the context of the 3-3-1 model.

The outline of the paper is the following. In Sec. II we introduce the interactions which are relevant to the present study. The model with $\langle \sigma_1^0 \rangle \neq 0$, which is some cases it has a Majoron-like Goldstone boson, is also discussed. In Sec. III we consider the more important contributions to the $(\beta\beta)_{0\nu}$ decay and the constraints upon some masses of the model. In Sec. IV we show that if we add a neutral scalar singlet to the minimal model a Majoron-like Goldstone boson is consistent with the Z^0 invisible width and we also discuss briefly the Majoron emission process $(\beta\beta)_{0\nu M}$ comparing the relative strength of two amplitudes. Our conclusions remain in the last section.

II. THE MODEL

Here we will consider the 3-3-1 model with the leptons belonging to triplets $(\nu_l \, l \, l^c)^T$; $l = e, \mu, \tau$ and in which a sextet of scalar bosons

$$S = \begin{pmatrix} \sigma_1^0 & \frac{h_2^-}{\sqrt{2}} & \frac{h_1^+}{\sqrt{2}} \\ \frac{h_2^-}{\sqrt{2}} & H_1^{--} & \frac{\sigma_2^0}{\sqrt{2}} \\ \frac{h_1^+}{\sqrt{2}} & \frac{\sigma_2^0}{\sqrt{2}} & H_2^{++} \end{pmatrix} \sim (\mathbf{6}, \mathbf{0}). \quad (3)$$

is necessary to give to the charged leptons a mass if $\langle \sigma_2^0 \rangle \equiv v_{\sigma_2} \neq 0$ [13].

Most of the phenomenological studies of the model has been done by considering $\langle \sigma_1^0 \rangle \equiv v_{\sigma_1} = 0$. The case when $\langle \sigma_1^0 \rangle \neq 0$ it was considered in Ref. [15], where the other scalar multiplets are explicitly given. The main difference in the latter case with respect to the former one is that there is a mixing between the vector bosons W^+ and V^+:

$$\begin{pmatrix} W_\mu^+ & V_\mu^+ \end{pmatrix} \begin{pmatrix} M_W^2 & \delta \\ \delta & M_V^2 \end{pmatrix} \begin{pmatrix} W_\mu^- \\ V_\mu^- \end{pmatrix}, \quad (4)$$

where $\delta = (g^2/2)(2v_{\sigma_1}v_{\sigma_2})$, M_W^2 and M_V^2 are the mass eigenvalues when $\delta = 0$; if $\delta \neq 0$ (i.e. when $v_{\sigma_1} \neq 0$) the mass of the physical fields are now

$$2M_{1,2} = (M_W^2 + M_V^2) \pm \left[(M_W^2 - M_V^2)^2 + 4\delta^2 \right]^{1/2} \quad (5)$$

and we have defined $M_W^2 = (g^2/2)(v_\eta^2 + v_\rho^2 + v_{\sigma_2}^2 + v_{\sigma_1}^2)$, $M_V^2 = (g^2/2)(v_\eta^2 + v_\chi^2 + v_{\sigma_2}^2 + v_{\sigma_1}^2)$ and g is the $SU(3)_L$ coupling constant which is numerically equal to the coupling of $SU(2)_L$ i.e., $g^2 = 8M_W^2 G_F/\sqrt{2}$. We have denoted by v_η, v_ρ and v_χ the vacuum expectation values of the neutral components of the triplets. Notice that $M_1 \to M_W$ and $M_2 \to M_V$ when $\delta \to 0$. The vector bosons W_μ^+

and V_μ^+ are related to the new mass eigenstates $W_{1\mu}^+$ and $W_{2\mu}^+$ as

$$\begin{pmatrix} W^+ \\ V^+ \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} W_1^+ \\ W_2^+ \end{pmatrix} \quad (6)$$

with $\tan 2\theta = -2\delta/(M_W^2 - M_V^2)$. We can obtain an upper bound on δ by assuming that the main contribution to the M_W^2 mass is given by $v_{\sigma_2} \approx 246$ GeV and that v_{σ_1} has its maximum value 3.89 GeV allowed by the value of the ρ-parameter [16]. In fact if v_{σ_2} were the main contribution to the M_W mass we would have $\delta/M_W^2 \approx (2v_{\sigma_1}/v_{\sigma_2}) < 0.032$. The constraint on the mixing angle θ is:

$$0 \leq s_\theta^2 = \frac{1}{2} \left(1 - \frac{M_V^2 - M_W^2}{[4\delta^2 + (M_V^2 - M_W^2)^2]^{1/2}} \right) < \frac{1}{2}. \quad (7)$$

Some illustrative values for s_θ are obtained by using typical values for the parameters. For instance, for $v_{\sigma_1} = 3.89$ GeV; $v_{\sigma_2} = 10$ GeV, $M_W = 80.41$ GeV and $M_V = 100\,(300)$ GeV we get $s_\theta^2 = 1.9 \times 10^{-5}(3.4 \times 10^{-8})$; or if $M_V = 100$ GeV and if v_{σ_2} has its maximal value $v_{\sigma_2} = 246$ GeV we have $s_\theta^2 = 1.1 \times 10^{-2}$. We see that only for values of $M_V \approx M_W$ the s_θ^2 is almost 0.5 but this light vector boson may be not phenomenologically safe. However if v_{σ_1} is of the same order of magnitude of the neutrino mass smaller values for the mixing angle are obtained. Hence, it may be no relevant for the collider physics and low energy processes like the $(\beta\beta)_{0\nu}$ decay at all and in practice $W_1^+ \approx W^+$, $W_2^+ \approx V^+$; but this could not be the case in astrophysical processes [15].

Next, we consider the several interactions that are present in this model. The scalar-quark interactions are

$$-\mathcal{L}_Y^{u-d} = \frac{\sqrt{2}}{|v_\rho|} \overline{D}_L V_{\text{CKM}}^\dagger M^u U_R \, \rho^- + \frac{\sqrt{2}}{|v_\eta|} \overline{U}_L V_{\text{CKM}} M^d D_R \, \eta_1^+$$

$$+ \overline{D}_L \left(V_L^d \right)^T \triangle V_L^u M^u U_R \left[\frac{\sqrt{2}}{|v_\eta|} \eta_1^- - \frac{\sqrt{2}}{|v_\rho|} \rho^- \right]$$

$$+ \overline{U}_L \left(V_L^u \right)^T \triangle V_L^d M^d D_R \left[\frac{\sqrt{2}}{|v_\rho|} \rho^+ - \frac{\sqrt{2}}{|v_\eta|} \eta_1^+ \right]$$

$$+ H.c., \quad (8)$$

with

$$\triangle \equiv \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (9)$$

and $V_L^{u,d}$ are unitary mixing matrices, and $M^{u,d}$ are the diagonal mass matrices of the u-like and d-like quark sectors, and V_{CKM} denotes the usual mixing matrix of Cabibbo-Kobayashi-Maskawa.

The Yukawa interactions in the lepton sector are

$$-\mathcal{L}_Y^l = \frac{1}{\sqrt{2}} \overline{\nu_L} \mathcal{K}_1 l_R H_1^+ + \frac{1}{\sqrt{2}} \bar{l}_L \mathcal{K}_2 \nu_R^c H_2^-$$

$$+ \frac{1}{2} \overline{l_L} \mathcal{K}_3 (l_L)^c H_1^{--} + \frac{1}{2} \overline{(l^c)_L} \mathcal{K}_4 l_R H_2^{++}$$
$$+ 2 \overline{\nu_L} \mathcal{K}_1' l_R \eta_1^+ - 2 \overline{l_L} \mathcal{K}_2' \nu_R^c \eta_2^- + H.c., \qquad (10)$$

where $\mathcal{K}_1 = E_L^{\nu\dagger} G E_R$, $\mathcal{K}_2 = E_L^\dagger G E_L^{\nu*}$; $\mathcal{K}_3 = E_L^\dagger G E_L$, $\mathcal{K}_4 = E_R^T G E_R$; $\mathcal{K}_1' = E_L^{\nu\dagger} G' E_R$, $\mathcal{K}_2' = E_L^\dagger G' E_L^{\nu*}$; G and G' are symmetric and antisymmetric (they can be complex) matrices, respectively. E_R, E_L, E_L^ν are the right- and left-handed mixing unitary matrices in the lepton sector relating symmetry eigenstates (primed fields) with mass eigenstates (unprimed fields) [17]:

$$l_R' = E_R l_R, \quad l_L' = E_L l_L, \quad \nu_L' = E_L^\nu \nu_L, \qquad (11)$$

Some of the couplings in Eq. (10) do not depend on the charged lepton masses and since all matrices in Eq. (10) are not unitary, the model breaks the lepton universality but it can be shown that, for the massless neutrino case no strong constraints arise from exotic muon and tau decays [18].

In the scalar sector we have also mixing angles. In the singly charged sector we have $\phi_i^- = \sum_l O_{ij} H_j^-$, where $\phi_i^- = \eta_1^-, \eta_2^-, \rho^-, \chi^-, h_1^-, h_2^-$ and H_j^-, $j = 1, ..., 6$ denotes the respective mass eigenstate field; similarly in the doubly charged sector we have $\Phi_i^{--} = \sum_l O_{ij} \Psi_j^{--}$, with $\Psi_i^{--} = \rho^{--}, \chi^{--}, H_1^{--}, H_2^{--}$ and Ψ_j^{--}, $j = 1, .., 4$ the respective mass eigenstate fields. However, in the following we will use H^- and H^{--} as typical mass eigenstates of the respective charged fields and omit the scalar mixing parameters.

We recall that the model conserves the $\mathcal{F} = L + B$ quantum number; L is the total lepton number and B is the baryon number. The assignments are:

$$\mathcal{F}(U^{--}) = \mathcal{F}(V^-) = \mathcal{F}(\rho^{--}) = \mathcal{F}(\chi^{--}) = \mathcal{F}(\eta_2^-)$$
$$= \mathcal{F}(\chi^-) = \mathcal{F}(\sigma_1^0) = \mathcal{F}(h_2^-) = \mathcal{F}(H_1^{--})$$
$$= \mathcal{F}(H_2^{--}) = 2, \qquad (12)$$

and all other scalar fields with $\mathcal{F} = 0$.

The charged currents coupled to the vector bosons are given by

$$\mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} \left(\overline{U}_L \gamma^u V_{\text{CKM}} D_L W_\mu^+ - \bar{\nu}_L \gamma^\mu V_W l_L W_\mu^+ \right.$$
$$\left. + \overline{l_L^c} \gamma^\mu V_U V_W^\dagger \nu_L V_\mu^+ - \overline{l_L^c} \gamma^\mu V_U l_L U_\mu^{++} \right) + H.c, \quad (13)$$

with the mixing matrices defined as $V_{\text{CKM}} = (V_L^u)^\dagger V_L^d$ in the quark sector; and $V_W = E_L^{\nu\dagger} E_L$, $V_U = E_R^T E_L^\nu$ in the leptonic sectors.

We have the trilinear interactions involving one vector- and two scalar-bosons which are of the form (up to a $ig/\sqrt{2}$ factor):

$$\mathcal{L}^{V2S} = \partial^\mu \chi^+ \chi^{--} W_\mu^+ + \chi^- \partial^\mu \chi^{++} W_\mu^- + \partial^\mu h_2^+ H_1^{--} W_\mu^+$$
$$+ \partial^\mu H_1^{++} h_2^- W_\mu^- + \partial^\mu \rho^- \rho^{++} V_\mu^- + \partial^\mu \rho^{--} \rho^+ V_\mu^+$$
$$+ h_1^+ \partial^\mu H_2^{--} V_\mu^+ + \partial^\mu h_1^- H_2^{++} V_\mu^- + \left(\eta_2^- \partial^\mu \eta_1^+ \right.$$
$$\left. + \eta_1^+ \partial^\mu \eta_2^+ + h_2^+ \partial^\mu h_1^+ + h_1^+ \partial^\mu h_2^+ \right) U_\mu^{--} + H.c.. \quad (14)$$

There are also trilinear interactions involving two vector- and one scalar-bosons (the χ^- scalar couples to ordinary and exotic quarks and for this reason it is not of our concern here). They are given by (up to a $g^2/2$ factor)

$$\mathcal{L}^{2VS} = \frac{v_{\sigma_1}}{2} H_1^{++} W^- \cdot W^- + \frac{v_{\sigma_1}}{2} H_2^{++} V^- \cdot V^-$$
$$+ \left(v_\rho \rho^{++} + v_\chi \chi^{++} + \frac{v_{\sigma_2}}{2} (H_1^{++} + H_2^{++}) \right)$$
$$W^- \cdot V^-. \qquad (15)$$

Notice that there is a coupling which is proportional to v_χ and hence it will be the dominant one.

Next, we write down the trilinear interactions among three vector bosons

$$\mathcal{L}^{3V} = i \frac{g}{\sqrt{2}} \left(W_{\mu\nu}^+ V^{\nu+} U^{\nu--} + V_{\mu\nu}^+ V^{\nu+} U^{\mu--} \right.$$
$$\left. + W^{\nu+} V^{\mu+} U_{\mu\nu}^{--} \right) \qquad (16)$$

where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ and so on.

Finally, we have trilinear interactions among scalar-bosons only

$$\mathcal{L}^{3S} = \frac{f_1}{2} \epsilon^{ijk} \eta_i \rho_j \chi_k + \frac{f_2}{2} \chi^T S^\dagger \rho + H.c.. \qquad (17)$$

The couplings $f_{1,2}$ have dimension of mass but are both of them arbitrary parameters (see next section). Other terms like the trilinears $f_3 \eta^T S^\dagger \eta$ and $f_4 \epsilon SSS$ and the quartic interactions $\epsilon \chi (S\eta^*) \rho$, $\chi^\dagger \eta \rho^\dagger \eta$ and $\epsilon \chi \rho SS$ violate the conservation of $\mathcal{F}$.

However, as we will show in Sec. IV, when discussing the Majoron emission, the model must be modified by adding a scalar singlet in order to be consistent with the LEP data.

III. THE NEUTRINOLESS DOUBLE BETA DECAY

Some of the more relevant diagrams of the $(\beta\beta)_{0\nu}$ decay in the present model are shown in Figs. 1-6. Our goal is to analyze the order of magnitude of each diagram and to obtain constraints on some mass scales of the model. We will consider the diagram in Fig. 1 as the reference one, i.e., it is the diagram that already exist in the standard model framework with massive Majorana neutrinos and which is parameterized by two effective four-fermion interactions. The other contributions will be considered as been at most equally important than the standard one.

The strength of the diagram in Fig. 1 is given by

$$A(1) \propto \frac{g^4 \langle M_\nu \rangle}{M_W^4 \langle p^2 \rangle} c_\theta^4 = \frac{32 G_F^2 \langle M_\nu \rangle}{\langle p^2 \rangle} c_\theta^4, \qquad (18)$$

where $\langle M_\nu \rangle$ is the effective mass defined in Eq. (1) and $\langle p^2 \rangle$ is the average of the four-momentum transfer
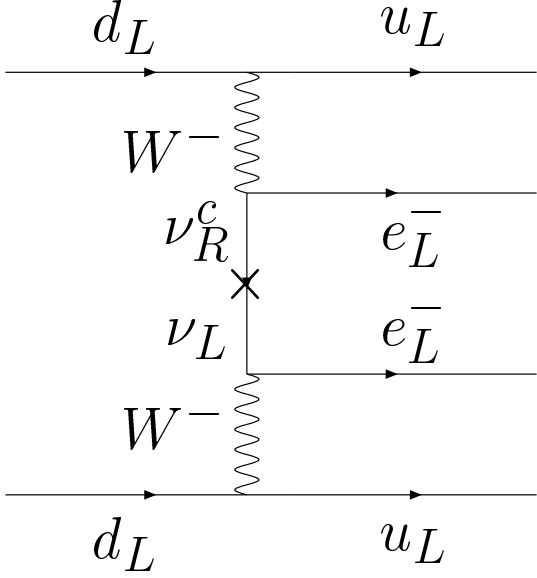
FIG. 1. $(\beta\beta)_{0\nu}$ decay via light massive Majorana neutrinos.

squared, which is of the order of $(100\,\mathrm{MeV})^2$. Below we will use a small δ so that $M_1 \to M_W$ and $M_2 \to M_V$.

In Eq. (18) and hereafter we will omit for simplicity the mixing parameters. Only in the vertices we will take care about the mixing between W and V defined in Eq. (6) but in the propagator we will use the masses of W and V.
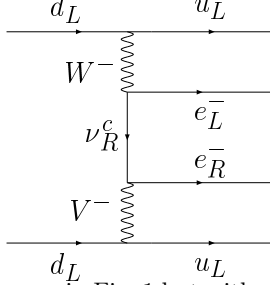

FIG. 2. The same as in Fig. 1 but with one bilepton vector boson V^- replaced by a vector boson W^-.

Next, let us consider the diagram in Fig. 2 which has the strength given by

$$A(2) \propto 32 G_F^2 \left(\frac{M_W}{M_V}\right)^2 \frac{c_\theta^3 s_\theta}{\sqrt{\langle p^2 \rangle}}, \qquad (19)$$

and we have the ratio

$$\frac{A(2)}{A(1)} = \left(\frac{M_W}{M_V}\right)^2 \frac{\sqrt{\langle p^2 \rangle}}{\langle M_\nu \rangle} \tan\theta, \qquad (20)$$

and if $A(2)/A(1) < 1$ we have that

$$M_V > 2.2 \times 10^4 M_W \sqrt{\tan_\theta} = 1.79 \times 10^6 \sqrt{\tan_\theta}\,\mathrm{GeV}. \qquad (21)$$

We recall that a lower limit of 440 GeV is obtained for M_V from the muon decays but when only the bilepton contributions to those decays are considered [19]. However, in the minimal 3-3-1 model the scalar-boson contributions cannot be negligible since some of the charged scalar-bosons can be lighter than the vector bilepton boson V^-. Hence, a lighter vector boson V may still be possible but this subject deserves a more detailed study of the muon decay considering both vector and scalar contributions. A contribution similar to that in Fig. 1 but with two V^- bosons instead of two W^- bosons may be not negligible but it does not constraint the mass M_V as much as those in Eq. (21) since the condition that its ratio to the $A(1)$ amplitude be less than one gives the condition $M_V > M_W \sqrt{\tan\theta}$.

All the Lagrangian interactions in Eqs. (8), (10), (13), (14), (15) and (16) are written in terms of symmetry eigenstates. We have assumed Yukawa couplings of the order of unity. As we are not considering the mixing among the scalar fields our constraints are valid only for the main component of the symmetry eigenstate scalar fields. It means that H^- and H^{--} denote the dominant mass eigenstates of the singly and doubly charged scalar fields, respectively.

The amplitude of the diagram in Fig. 3 is

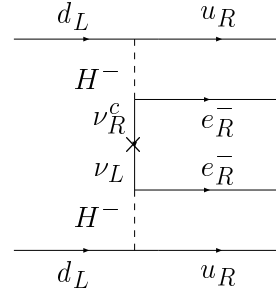$$A(3) \propto \frac{\langle M_\nu \rangle}{\langle p^2 \rangle M_{H^-}^4}. \qquad (22)$$

FIG. 3. Charged scalars contribution to the $(\beta\beta)_{0\nu}$ decay.

The scalar contribution in Fig. 3 can be as important as the standard one in Fig. 1. We have

$$\frac{A(3)}{A(1)} = \frac{1}{32 G_F^2 M_{H^-}^4 \, c_\theta^4}, \qquad (23)$$

and assuming that $A(3)/A(1) < 1$ and $c_\theta = 1$ we get

$$M_{H^-} > 124\,\mathrm{GeV}. \qquad (24)$$

From Eq. (15) we see that the contribution $v_\chi \chi^{++} W_\mu^- V^{\mu-}$ is the dominant one in diagrams like that in Fig. 4. As we said before we will omit the mixing angles, i.e., assuming $\chi^{--} \approx H^{--}$. Hence we have

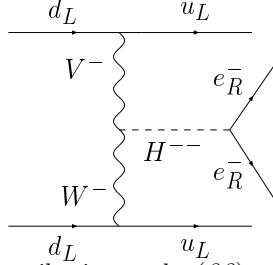$$A(4) \propto \frac{v_\chi}{M_W^2 M_V^2 M_{H^{--}}^2} c_\theta^2 s_\theta^2. \qquad (25)$$

FIG. 4. New contribution to the $(\beta\beta)_{0\nu}$ decay in the 3-3-1 model.

Next, we note that

$$\frac{A(4)}{A(1)} \propto \frac{v_\chi}{\langle M_\nu \rangle} \frac{\langle p^2 \rangle}{32 G_F^2 M_W^2 M_V^2 M_{H^{--}}^2} \tan^2 \theta,$$

$$\approx 5.33 \times 10^{15} \tan^2 \theta \frac{(1\text{GeV})^4}{M_V^2 M_{H^{--}}^2}, \qquad (26)$$

where we used $\langle M_\nu \rangle = 0.2$ eV [4], $v_\chi = 3$ TeV and $\langle p^2 \rangle = (100\,\text{MeV})^2$. If $A(4)/A(1) < 1$ it implies

$$M_V > 7.3 \times 10^7 \tan \theta \frac{(1\text{GeV})^2}{M_{H^{--}}}. \qquad (27)$$

Similar analysis arises by considering Fig. 5, however it is less enhanced than the contribution of Fig. 4 because instead of v_χ it appears the momentum of one of the vector bosons, $p \sim \sqrt{\langle p^2 \rangle}$.
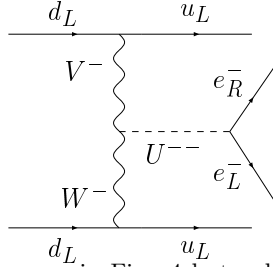

FIG. 5. The same as in Fig. 4 but only involving vector bosons.

More interesting are the contributions involving trilinear scalar interactions given in Eq. (17) like that of the diagram in Fig. 6. We have in this case

$$A(6) \propto \frac{f}{M_{H^-}^4 M_{H^{--}}^2}, \qquad (28)$$

where M_{H^-} represents a typical mass of the singly charged scalar bosons, say 124 GeV; $M_{H^{--}}$ is the mass of the doubly charged scalar boson and f is the trilinear coupling f_1 or f_2 in Eq. (17) with dimension of mass. The ratio of these amplitudes is:

$$\frac{A(6)}{A(1)} \propto \frac{f \langle p^2 \rangle}{32 G_F^2 M_{H^-}^4 M_{H^{--}}^2 \langle M_\nu \rangle c_\theta^4}$$

$$\approx \left(\frac{f/\text{GeV}}{M_{H^{--}}^2/\text{GeV}^2} \right) \cdot \frac{4.8 \times 10^7}{c_\theta^4}, \qquad (29)$$

If $A(6)/A(1) < 1$, and assuming $c_\theta = 1$ and $M_{H^-} = 124$ GeV, we obtain the constraint

$$\frac{f}{M_{H^{--}}^2} < 2.1 \times 10^{-8}\,\text{GeV}^{-1}. \qquad (30)$$

For arbitrary $U(1)_N$ charge for the scalar multiplets the symmetry of the potential is $SU(3)_L \otimes [U(1)]^2$. If the triplet η and the sextet S have both $N = 0$, as it is the case for the present model, the trilinear couplings $f_{1,2}$ break the extra $U(1)$ symmetry. We have verify that if both $f_{1,2} = 0$ there is indeed a pseudo-Goldstone boson [20]. It means that $f_{1,2}$ are arbitrary parameters and in principle they can be small (say 1 GeV), or large (say 1 TeV) mass scales.

We see that if $f = 1$ (10^{-3}) TeV then $M_{H^{--}}$ is greater or of the order of 300 (10) TeV. For this value for the mass of the doubly charged scalar field and θ small the constraint given in Eq. (21) is stronger than that of Eq. (27). For instance if $\tan \theta = 10^{-8}$ we have from Eq. (21) that $M_V \geq 179$ GeV.
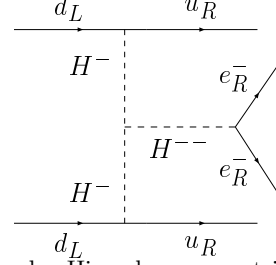

FIG. 6. Pure scalar Higgs bosons contribution the $(\beta\beta)_{0\nu}$ decay in the 3-3-1 model.

There is also a diagram in which the doubly charged scalar field in Fig. 6 is substituted by a vector boson U^{--}. Although the interactions in Eq. (14) are proportional to g they are also derivative and proportional to the momentum $p \sim \sqrt{\langle p^2 \rangle}$; hence it is suppressed with respect to the diagram in Fig. 6.

IV. MAJORON EMISSION

If the $\mathcal{F}$ quantum number is spontaneously broken as in the present model, it means that a Majoron-like bosons does exist. Since the scalar field that is responsible for the breakdown of that continuous symmetry is σ_1^0, and it belongs to a triplet of the subgroup $SU(2) \otimes U(1)$, this Majoron-like Goldstone has similar couplings that the triplet Majoron model of Ref. [21]. It is well known that this sort of Majoron model has been ruled out by the LEP data [22]. Apparently, since the Higgs sector of the present model is rather complicated having a neutral scalar singlet (under $SU(2) \otimes U(1)$), χ^0, it seems that the Majoron-like Goldstone in this case it will be able to avoid the LEP constraints as claimed in Refs. [15,23]. However, we will show that this is not indeed the case.

The mass matrices of the scalar and pseudoscalar in this model have been given in Ref. [15]. Here we will only give the results of the mass eigenvalues and the respective mixing matrix in the CP-even scalar sector. The argument in Ref. [15] was the following. Let us begin with the relation $R_4^0 = \sum_j \mathcal{O}_{4j}^o H_j^0$, where H_j^0, $j = 1, ...5$, denotes the mass scalar eigenstates and R_4 the real component of the scalar field σ_1^0 according to the general shifting of the neutral scalar fields in the scalar potential of the form $X_i^0 \to (1/\sqrt{2})(v_{X_i} + R_i + iI_i)$, $i = 1, 2, 3, 4, 5$ where $X_i^0 = \eta, \rho, \chi, \sigma_1, \sigma_2$ respectively. In this case if H_1^0 denotes the lightest scalar boson ($M_{H_1} < M_Z$, we are assuming a mass spectrum where $M_{H_i} < M_{H_j}$ if $i < j$), the contribution to the decay mode $Z^0 \to H_1^0 M^0$ is $\Gamma_{H_1^0 M^0}^Z = 2|\mathcal{O}_{41}^o|^2 \Gamma_{\nu\bar{\nu}}^Z$. Hence, if $|\mathcal{O}_{41}^o| < 10^{-2}$ the model would be consistent with the LEP data i.e., now $\Gamma_{H_1^0 M^0}^Z$ would be reduced to an acceptable level.

First of all recall that as shown in Ref. [15] the Majoron-like boson decouples from the other pseudoscalar fields i.e., $\text{Im}\sigma_1^0 \equiv I_4 = M^0$. For instance, using the same values of the VEV and $f_{1,2} = -1$ TeV and with the dimensionless constant of the scalar potential given in Ref. [15] with $\lambda_k = 0.1$ for $k = 1, 2, 3, 4, 5, 6, 7, 8, 9, 18, 20$, $\lambda_m = 0.01$ for $m = 13, 14, 16, 17$, $\lambda_n = 0.001$ for $n = 10, 11, 12, 19$ and $\lambda_{15} = 0.05$ we obtain the following masses in the scalar sector $(0.056, 102, 1342, 3626, 4325)$ GeV and the mixing matrix (up to three decimal places)

$$O^o = \begin{pmatrix} 0.0 & 0.081 & -0.010 & 0.996 & 0.021 \\ 0.0 & 0.995 & -0.029 & -0.082 & 0.039 \\ 0.0 & -0.030 & -0.999 & -0.008 & 0.004 \\ 1.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.040 & -0.005 & 0.017 & -0.999 \end{pmatrix} \quad (31)$$

This pattern of mixing matrix remains the same for several values of the parameters provided that v_{σ_1} is a small VEV restricted to the condition that it has to be smaller than 3.89 GeV [16]. From Eq. (31) it can be seen that the scalar partner of the Majoron is always mainly the lightest scalar i.e., $|\mathcal{O}_{41}^o| \sim 1$ and it would be always produced at LEP. We see that the Majoron in the minimal 3-3-1 model has been also ruled out by the LEP data.

One possibility to recover consistency with the LEP data is to break explicitly the $\mathcal{F}$ symmetry by adding trilinear terms like $f_3\eta^T S\eta$, $f_4\chi^T S^\dagger\rho$ in the scalar potential, see Eq. (17). In this case there is no Majoron at all and although v_{σ_1} still has a small value, due to f_3 all scalars are heavy enough to not be produced at the LEP energies [24,25]. Of course, in this case there is no contribution to Majoron emission in the neutrinoless double beta decay. However, our results in Sec. III are still valid since they depend only on the small value of v_{σ_1}.

Another possibility which we will consider here is to modify the model by introducing a scalar singlet Σ^0, which carries $\mathcal{F} = 2$ (or $L = 2$), in the same way as considered in Ref. [26,27] in the context of a $SU(2) \otimes U(1)$ model. In this case we have to add the following terms to the scalar potential in Ref. [15]

$$V(X_i, \Sigma) = \mu_5^2 \Sigma^2 + \lambda_{21}\Sigma^4 + \sum_i \Big[\lambda_{X_i} Tr\left(X_i^\dagger X_i\right)\Sigma^2$$
$$- \kappa\eta^T S^\dagger \eta\Sigma + H.c.\Big] \quad (32)$$

where X_i denotes any triplet η, ρ, χ or the sextet S, and we will denote λ_{X_i} as $\lambda_{22,23,24,25}$ respectively and $\kappa > 0$. The neutral Higgs sector contains six CP-even scalars and three massive CP-odd pseudoscalar beside the massless CP-odd Majoron. The neutral scalar singlet also gains a VEV, i.e., $\Sigma = (v_\Sigma + R_6 + iI_6)/\sqrt{2}$, and the mass term is given by $M^2/2$, where M^2 in the pseudoscalar sector in the basis $I_1, I_2, I_3, I_4, I_5, I_6$ is given by (the constraints equation appear in the Appendix)

$$M_{11} = -\frac{\lambda_{16}}{2\sqrt{2}}\frac{v_\rho^2 v_{\sigma_2}}{v_\eta} + 2\lambda_{17}v_{\sigma_2}^2 + \frac{1}{2\sqrt{2}}(\lambda_{15}v_\chi v_{\sigma_2}$$
$$- f_1 v_\rho)\frac{v_\chi}{v_\eta} + 2\kappa v_{\sigma_1}v_\Sigma + \frac{t_\eta}{v_\eta},$$
$$M_{22} = -\frac{1}{4}(\sqrt{2}f_1 v_\eta + f_2 v_{\sigma_2})\frac{v_\chi}{v_\rho} + \frac{t_\rho}{v_\rho},$$
$$M_{33} = -\frac{1}{4}(\sqrt{2}f_1 v_\eta + f_2 v_{\sigma_2})\frac{v_\rho}{v_\chi} + \frac{t_\chi}{v_\chi},$$
$$M_{44} = \kappa\frac{v_\eta^2 v_\Sigma}{2v_{\sigma_1}} + \frac{t_{\sigma_1}}{v_{\sigma_1}},$$
$$M_{55} = \frac{1}{2\sqrt{2}}(\lambda_{15}v_\chi^2 - \lambda_{16}v_\rho^2)\frac{v_\eta}{v_{\sigma_2}} + 2\lambda_{17}v_\eta^2 - \frac{f_2}{4}\frac{v_\rho v_\chi}{v_{\sigma_2}}$$
$$+ \frac{t_{\sigma_2}}{v_{\sigma_2}}, \quad M_{66} = \kappa\frac{v_\eta^2 v_{\sigma_1}}{2v_\Sigma} + \frac{t_\Sigma}{v_\Sigma},$$
$$M_{12} = -\frac{f_1}{2\sqrt{2}}v_\chi, \quad M_{13} = -\frac{f_1}{2\sqrt{2}}v_\rho, \quad M_{14} = -\kappa v_\eta v_\Sigma,$$
$$M_{15} = \frac{1}{2\sqrt{2}}(\lambda_{15}v_\chi^2 - \lambda_{16}v_\rho^2) + 2\lambda_{17}v_\eta v_{\sigma_2}, \quad M_{16} = \kappa v_\eta v_{\sigma_1},$$
$$M_{23} = -\frac{1}{4}(\sqrt{2}f_1 v_\eta + f_2 v_{\sigma_2}), \quad M_{24} = 0, \quad M_{25} = \frac{f_2}{4}v_\chi,$$
$$M_{26} = 0, \quad M_{34} = 0, \quad M_{35} = \frac{f_2}{4}v_\rho, \quad M_{36} = 0,$$
$$M_{45} = 0, \quad M_{46} = -\kappa\frac{v_\eta^2}{2}, \quad M_{56} = 0. \quad (33)$$

The mass matrix above has two true Goldstone bosons $G_{1,2}^0$ and the Majoron-like one, M^0, and three massive CP-odd pseudoscalar bosons. The massless ones are given by

$$G_1^0 = (0, v_\rho/v_\chi, -1, 0, 0, 0)/(1 + v_\rho^2/v_\chi^2)^{1/2}$$
$$G_2^0 = (\frac{v_\eta}{v_{\sigma_2}}, -\frac{v_\rho v_\chi^2}{V_1}, -\frac{v_\rho^2 v_\chi}{V_1}, \frac{2v_{\sigma_1}v_\Sigma^2}{V_2}, -1, -\frac{2v_{\sigma_1}^2 v_\Sigma}{V_2})/N$$
$$M^0 = (0, 0, 0, v_{\sigma_1}/v_\Sigma, 0, 1)/(1 + v_{\sigma_1}^2/v_\Sigma^2)^{1/2} \quad (34)$$

where $V_1 = v_{\sigma_2}(v_\rho^2 + v_\chi^2)$, $V_2 = v_{\sigma_2}(v_{\sigma_1}^2 + v_\Sigma^2)$; N is the normalization factor that we will omit here. We have verify that M^0 in Eq. (34) is in fact the Majoron: by adding an explicit $\mathcal{F}$-violating term, like $f_3\eta^T S\eta$, it gets a mass

while the other two $G_{1,2}$ remain massless. The massive pseudoscalars, for the parameters used before have the following masses (in GeV): 174, 3625 and 4325. On the other hand, if $v_{\sigma_1} = 0$, which forces $\kappa = 0$, the Majoron is purely singlet and the real and imaginary parts of σ_1^0 are mass degenerate, i.e., form a complex field, with mass

$$m_{\sigma_1} = \mu_4^2 + \lambda_{10}v_{\sigma_2}^2 + (\lambda_{12} + \lambda_{19})\frac{v_\eta^2}{2} + \lambda_{13}\frac{v_\chi^2}{2}$$
$$+ \lambda_{14}\frac{v_\rho^2}{2} + \lambda_{25}\frac{v_\Sigma^2}{2}. \tag{35}$$

We see that in this case the Majoron has not doublet components at all and it is mainly singlet. Hence it is possible to keep consistence with LEP data. Although there are astrophysical constraints (the Majoron emission implies a different rate for the stellar cooling) that have to be taken into account [28], in the basis we have chosen they are less severe since we have avoided the doublet component of the Majoron. Any way, since these constraints have been already considered in Ref. [27] and they imply that $v_{\sigma_1} < 0.33$ GeV if $v_\Sigma = 1$ TeV, we will use these values for v_{σ_1}, v_Σ.

Once we have shown in what situation there is a safe Majoron-like boson in the present model we can consider the emission of this Goldstone boson in the neutrinoless double beta decay. In fact, as in the triplet Majoron model, in the present model it is possible to have the neutrinoless double beta decay with Majoron emission: $2n \to 2p + 2e^- + M^0$ [29], denoted here by $(\beta\beta)_{0\nu M}$. We will denote the strength of the amplitude of the diagram i of the $(\beta\beta)_{0\nu M}$ decay by $B(i)$. This decay proceeds via the diagram in Fig. 7 and it has an strength proportional to

$$B(7) \propto \frac{m_\nu(v_{\sigma_1}/v_\Sigma)}{M_{X^-}^4\langle p^2\rangle v_{\sigma_1}}, \tag{36}$$

where X^- can be an scalar or a vector boson, i.e., the diagram in Fig. 7 can be formed with anyone of Figs.1, 2 or 3 with a Majoron attached to the neutrinos. The couplings between neutrinos and the Majoron are diagonal and given by m_ν/v_{σ_1}. Notice that in Eq. (36) it appears the truly neutrino mass instead of the effective mass $\langle M_\nu\rangle$ defined in Eq. (1). However we still can assume that neutrinos have small masses and numerically $m_\nu \approx \langle M_\nu\rangle$. We will assume also that the contribution to the $(\beta\beta)_{0\nu M}$-decay in Fig. 7 with $X^- = W^-$ is the reference one. This diagram depends only on the neutrino masses and mixing angles, and we will compare it with other contributions like the one in Fig. 8. The couplings of the Majoron to the vector bosons are proportional to v_{σ_1} and so they are negligible. We will consider only the diagram with the Majoron coupled to the scalar H^- since it is proportional to the trilinear f_2 shown in Eq. (17). We have

$$B(8) \propto \frac{f f_2}{M_{H^-}^6 M_{H^{--}}^2}, \tag{37}$$

with f can be f_1 or f_2.

Let us consider the ratio

$$\frac{B(7)}{A(1)} \propto \frac{m_\nu Q}{32G_F^2 M_X^4\langle M_\nu\rangle c_\theta^4 v_\Sigma}, \tag{38}$$

where we have introduced the factor Q which denotes the available energy. It implies that the diagram in Fig. 7 is a potentially important contribution when X is the W vector boson since for $Q \sim 3$ MeV [30] the suppression of $B(7)$ will depend mainly on the value of v_Σ. If $B(7)/A(1) < 1$ we obtain that $v_\Sigma > 1.65 \times 10^{-2}$ GeV which is automatically satisfied.
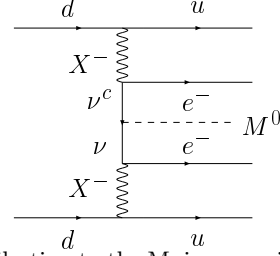

FIG. 7. Contribution to the Majoron emission $(\beta\beta)_{0\nu M}$ decay. X^- can be a scalar or vector boson.

On the other hand, comparing the amplitudes of the diagrams in Figs. 7 and 8 we have

$$\frac{B(8)}{B(7)} \propto \frac{f f_2\langle p^2\rangle M_X^4 v_\Sigma}{M_{H^-}^6 M_{H^{--}}^2 m_\nu}, \tag{39}$$

and for $M_X = M_V = 400$ GeV and $v_\Sigma = 1$ TeV, using typical values as $f = f_1 = f_2 = -1$ TeV, $M_{H^-} = 124$ GeV and $M_{H^{--}} = 500$ GeV, and the other parameters in Eq. (39), we have that $B(8)/B(7) \approx 1.3 \times 10^9$ or; $B(8)/B(7) \approx 1.5 \times 10^3$ if $f_1 = f_2 = f = -10^{-3}$ TeV. The relative importance of the processes in Fig. 7 and 8 will depend on the values of the trilinear parameters and on the value of v_Σ.
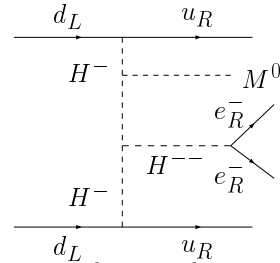

FIG. 8. Trilinear scalar coupling contributing to the $(\beta\beta)_{0\nu M}$ decay.

V. CONCLUSIONS

We see that in the 3-3-1 model, like in other models with complicated Higgs sector [8,12], besides the well known mechanism of exchanging massive Majorana neutrinos between two standard model $V - A$ vertices, there

are new contributions involving the exchange of scalar bosons. However, unlike similar mechanism in the context of extensions of the standard model there is no need of fine tuning in order to have trilinear scalar couplings giving large contributions to the several neutrinoless double beta decay modes. Notice that effective interactions from diagrams like those in Fig. 3 are still parameterized in the form of two general four-fermion effective interactions (they are point-like at the Fermi scale) exchanging a light neutrino in between [31]. However, contributions involving trilinear interactions like those in Figs. 4, 5 and 6 necessarily need a six-fermion effective interaction parameterization.

Another important point to be stressed here is that in the present model the double Majoron emission: $2d \rightarrow 2p + 2e^- + 2M^0$ may be as contributing as the decay with only one Majoron boson. This decay is expected to be important in supersymmetric models [32,33]. In the present model it can occur because in diagrams like that in Fig. 8 a second Majoron can be attached to the scalar lines. Since this coupling is proportional to the trilinear f_2 it is still possible that the suppression coming from the mass square in the denominator do not sufficiently suppresses this process (there is also an important contribution coming from the vertex $v_\chi \chi^- \eta_1^+ M^0$). There are also contributions similar to the one in Fig. 8 but now with the scalar-Majoron vertex above being substituted by the vertex $W^- V^+ M^0$ which is proportional to $g^2 v_{\sigma_2}$ and for this reason it is not necessarily suppressed. It is interesting to note that this sort of contribution to the $(\beta\beta)_{0\nu MM}$ decay, coming from adding another trilinear coupling in the diagram in Fig. 8, which is not derivatively suppressed, was not considered in Ref. [33]. Recent experimental data on Majoron emission decays have been constrained only the effective Majoron-neutrino coupling constant [30]. Other processes like the double K capture [29] can also be important in the present model.

Some comments are now in order. *i)* We have not considered possible cancellations among several contributions to each diagram. It means that our constraints are valid, as we said in Sec. III, for the main component of each scalar field of the singly and doubly charged scalar sectors.

ii) Our results were obtained assuming that all new contributions, say to the $(\beta\beta)_{0\nu}$ decay, are at most as important as the contribution due to a light massive Majorana neutrino exchange which is proportional to $\langle M_\nu \rangle$. However, we can wonder what would be the value of the effective mass $\langle M_\nu \rangle$ if we use the oscillation data and direct measurements on neutrino masses. Recent analysis shown that assuming a normal mass hierarchy the effective mass parameter can take any value from zero to the present upper limit [34]. In fact, if the data on oscillation is put together with that from $(\beta\beta)_{0\nu}$ decay and tritium beta decay, it was shown that if the minimum of $\langle M_\nu \rangle$ with respect to the mixing angles is greater than the present bound of 0.2 eV, then neutrinos are quasi-Dirac particles [35]. As discussed previously, the black box of Ref. [8] may induce a negligible Majorana mass to the neutrinos and in the context of the present model we must interpret this situation as an indication of the fact that the main contribution to the $(\beta\beta)_{0\nu}$ decay is not the diagram in Fig. 1. In this case the neutrinos would be almost Dirac particles and the constraints on the several mass scales of the model should be obtained by comparing directly these contributions with the lower bound on the half life $T_{1/2}^{0\nu} > 1.8 \times 10^{25}$ yr [7].

iii) In the basis we have chosen, see Eq. (34), the Majoron couples at the tree level only to neutrinos, and hence the constraints on the ν-ν-M vertex, coming from muon ($\mu \rightarrow e\nu\nu M$), pion and kaon $\pi^+(K^+) \rightarrow l\bar\nu M$ decays, are the same as in Ref. [33]. The existence of the vertex $\nu e^- V^+$, which is proportional to $\sin\theta$ and for this reason is not relevant for laboratory processes, may have, as we said before, important astrophysical consequences [15].

iv) phenomenology of non-zero initial electric charge processes, like $e^- e^-$ and hadronic ones, will furnish constraints on the trilinear vertices appearing in Figs. 4, 5 and 6 but this will be considered elsewhere. The 3-3-1 model has a rich scalar sector indeed. This implies that it may be, in principle, difficult to separate, in a given process, the contributions of all fields belonging to a charged sector. However, it has been shown that in lepton-lepton colliders the left-right asymmetries are not sensible to the scalar contributions. It means that those asymmetries are the appropriate observable for the doubly charged vector bilepton discovery [36]. We see that the opposite occurs in the $(\beta\beta)_{0\nu}$ decay: it is possible that the main contribution comes from the doubly charged scalar boson, through the diagram in Fig. 6, while the respective vector boson contribution seems to be negligible.

Finally we would like to compare our Majoron model with that of Schechter and Valle [26]. Firstly, we notice that although our model has two singlet, three doublets and a triplet of scalars under the subgroup $SU(2) \otimes U(1)$, the respective scalar potential is not reduced to the scalar potential invariant under the standard $SU(2) \otimes U(1)$ symmetry, involving the same multiplets. For instance in our model there are cubic invariants which are not present in the former. Secondly, we have not introduced right-handed neutrinos and for this reason we have only light neutrinos. It means that the singlet Σ does not couple to the leptons and that the coupling of neutrinos to Majoron and Z^0 are diagonal. Thus, the decays $\nu_H \rightarrow \nu_L + M^0$ and $\nu_H \rightarrow \nu_L + \nu'_L + \nu'_L$, are not induced at the tree level, where ν_H, although light, is heavier that ν_L. The decay $\nu_H \rightarrow \nu_L^c + M^0$ is produced at the one loop level due to the mixing between W and V. The vertex is proportional to $(g^2/\sqrt{2})(v_{\sigma_2} v_{\sigma_1}/v_\Sigma)$; then even with v_{σ_2} of the order of 10 GeV and v_Σ of the order of 1 TeV the lifetime is of the order of the age of the universe. (The decay $\nu_H \rightarrow \nu_L + M^0$ also occurs but the vertex involved is proportional to $(g^2/\sqrt{2})(v_{\sigma_1}^2/v_\Sigma)$.) Notice also that in the basis given in Eq. (34) the Majoron does not couple to the

charged leptons so there is not the process $\gamma + e \to M^0 + e$ at the tree level which imposes severe astrophysical constraints in v_{σ_1} as has been noted in Ref. [27].

ACKNOWLEDGMENTS

APPENDIX A: CONTRAINTS EQUATION OF THE SCALAR POTENTIAL

Here we show the constraints equation that must be satisfied by the scalar potential

$$t_\eta = \mu_1^2 v_\eta + \lambda_1 v_\eta^3 + \frac{\lambda_4}{2} v_\rho^2 v_\eta + \frac{\lambda_5}{2} v_\chi^2 v_\eta + \frac{\lambda_{12}}{2}(v_{\sigma_1}^2 + v_{\sigma_2}^2) v_\eta$$
$$- \frac{\lambda_{15}}{2\sqrt{2}} v_\chi^2 v_{\sigma_2} + \frac{\lambda_{16}}{2\sqrt{2}} v_\rho^2 v_{\sigma_2} - \lambda_{17} v_{\sigma_2}^2 v_\eta + \frac{\lambda_{19}}{2} v_{\sigma_1}^2 v_\eta$$
$$- \kappa v_\eta v_{\sigma_1} v_\Sigma + \frac{f_1}{2\sqrt{2}} v_\rho v_\chi,$$

$$t_\rho = \mu_2^2 v_\rho + \lambda_2 v_\rho^3 + \frac{\lambda_4}{2} v_\eta^2 v_\rho + \frac{\lambda_6}{2} v_\chi^2 v_\rho + \frac{\lambda_{14}}{2}(v_{\sigma_1}^2 + v_{\sigma_2}^2) v_\rho$$
$$+ \frac{\lambda_{16}}{\sqrt{2}} v_{\sigma_2} v_\eta v_\rho + \frac{\lambda_{20}}{4} v_{\sigma_2}^2 v_\rho + \lambda_{23} \frac{v_\rho v_\Sigma^2}{2}$$
$$+ \frac{f_1}{2\sqrt{2}} v_\eta v_\chi + \frac{f_2}{4} v_{\sigma_2} v_\chi,$$

$$t_\chi = \mu_3^2 v_\chi + \lambda_3 v_\chi^3 + \frac{\lambda_5}{2} v_\eta^2 v_\chi + \frac{\lambda_6}{2} v_\rho^2 v_\chi + \frac{\lambda_{13}}{2}(v_{\sigma_1}^2 + v_{\sigma_2}^2) v_\chi$$
$$- \frac{\lambda_{15}}{\sqrt{2}} v_{\sigma_2} v_\eta v_\chi + \frac{\lambda_{18}}{4} v_{\sigma_2}^2 v_\chi + \lambda_{24} \frac{v_\chi v_\Sigma^2}{2}$$
$$+ \frac{f_1}{2\sqrt{2}} v_\eta v_\rho + \frac{f_2}{4} v_{\sigma_2} v_\rho,$$

$$t_{\sigma_1} = \mu_4^2 v_{\sigma_1} + \lambda_{10}(v_{\sigma_1}^2 + v_{\sigma_2}^2) v_{\sigma_1} + \lambda_{11} v_{\sigma_1}^3 + \frac{\lambda_{12}}{2} v_\eta^2 v_{\sigma_1}$$
$$+ \frac{\lambda_{13}}{2} v_\chi^2 v_{\sigma_1} + \frac{\lambda_{14}}{2} v_\rho^2 v_{\sigma_1} + \frac{\lambda_{19}}{2} v_\eta^2 v_{\sigma_1} + \frac{\lambda_{25}}{2} v_{\sigma_1} v_\Sigma^2 - \frac{\kappa}{2} v_\eta^2 v_\Sigma,$$

$$t_{\sigma_2} = \mu_4^2 v_{\sigma_2} + \lambda_{10}(v_{\sigma_2}^2 + v_{\sigma_1}^2) v_{\sigma_2} + \frac{\lambda_{11}}{2} v_{\sigma_2}^3 + \frac{\lambda_{12}}{2} v_\eta^2 v_{\sigma_2}$$
$$+ \frac{\lambda_{13}}{2} v_\chi^2 v_{\sigma_2} + \frac{\lambda_{14}}{2} v_\rho^2 v_{\sigma_2} - \frac{\lambda_{15}}{2\sqrt{2}} v_\chi^2 v_\eta + \frac{\lambda_{16}}{2\sqrt{2}} v_\rho^2 v_\eta - \lambda_{17} v_\eta^2 v_{\sigma_2}$$
$$+ \frac{\lambda_{18}}{4} v_\chi^2 v_{\sigma_2} + \frac{\lambda_{20}}{4} v_\rho^2 v_{\sigma_2} + \frac{\lambda_{25}}{2} v_{\sigma_2} v_\Sigma^2 + \frac{f_2}{4} v_\rho v_\chi,$$

$$t_\Sigma = \mu_5^2 v_\Sigma + \lambda_{21} v_\Sigma^3 + \frac{\lambda_{22}}{2} v_\eta^2 v_\Sigma + \frac{\lambda_{23}}{2} v_\rho^2 v_\Sigma + \frac{\lambda_{24}}{2} v_\chi^2 v_\Sigma$$
$$+ \frac{\lambda_{25}}{2}(v_{\sigma_1}^2 + v_{\sigma_2}^2) v_\Sigma - \frac{\kappa}{2} v_\eta^2 v_\sigma.$$

[1] K. Lande (Homestake Collaboration), in *Neutrinos '98*, Proceedings of the XVIII International Conference on Neutrino Physics and Astrophysics, Japan, 4-9 June 1998, edited by Y. Suzuki and T. Tosuka, to be published in Nucl. Phys. B (Proc. Suppl.); Y. Fukuda *et al.* (Kamiokande Collaboration), Phys. Rev. Lett. **77**, 1683 (1996); T. Kirsten (GALLEX Collaboration) in *Neutrinos '98*; V. Gavrin (SAGE Collaboration) in *Neutrinos '98*; Y. Suzuki (SuperKamiokande Collaboration) in *Neutrinos '98*.

[2] Y. Fukuda *et al.* (SuperKamiokande Collaboration), Phys. Lett. **B433**, 9 (1998); Phys. Rev. Lett. **81**, 1562 (1998); Phys. Lett. **B436**, 33 (1988).

[3] C. Athanassopoulos *et al.*, Phys. Rev. Lett. **77**, 3082 (1996); *ibid*, **81**, 1774 (1998).

[4] D. E. Groom *et al.*, The European Physical Journal, **C15**, 1 (2000).

[5] H. Primakov and S. P. Rosen, Rep. Prog. Phys. **22**, 111 (1969).

[6] For a recent review see H. V. Klapdor-Kleingrothaus, Int. J. Mod. Phys. **A13**, 3953 (1998); P. Fisher, B. Kayser and K. S. McFarland, Ann. Rev. Nucl. Part. Sci. **49** (1999); hep-ph/9906244. For early references see M. Doi, T. Kotani and E. Takasugi, Progr. Theor. Phys. Suppl. **83**, 1 (1985).

[7] L. Baudis *et al.*, Phys. Rev. Lett. **83**, 41 (1999); hep-ex/9902014.

[8] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 2951 (1982).

[9] V. Pleitez and M. D. Tonasse, Phys. Rev. D **48**, 527 (1993).

[10] R. N. Mohapatra and J. D. Vergados, Phys. Rev. Lett. **47**, 1713 (1981).

[11] W. C. Haxton, S. P. Rosen and G. J. Stephenso, Jr., Phys. Rev. D **26** 1805 (1982); L. Wolfenstein, *ibid*, D **26**, 2507 (1982).

[12] C. O. Escobar and V. Pleitez, Phys. Rev. D **28**, 1166 (1983).

[13] F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992); P. Frampton, Phys. Rev. Lett. **69**, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D **47**, 4158 (1993).

[14] M. Hirsch, H. V. Kkapdov-Kleingrothaus and S. G. Kovalenko, Phys. Rev. Lett. **75**, 17 (1995); Phys. Rev. D **53**, 1329 (1996); H. V. Klapdor-Kleingrothaus and H. Päs, hep-ph/0005045.

[15] J. C. Montero, C. A. de S. Pires and V. Pleitez, Phys. Rev. D **60**, 115003 (1999); hep-ph/9903251.

[16] J. C. Montero, C. A. de S. Pires and V. Pleitez, Phy. Rev. D **60**, 98701 (1999); hep-ph/9902448.

[17] J. T. Liu and D. Ng, Phys. Rev. D **50**, 548 (1994).

[18] M. M. Guzzo et al., hep-ph/9908308.

[19] M. B. Tully and G. C. Joshi, Int. J. Mod. Phys. **A13**, 5593 (1998).

[20] S. Weinberg, Phys. Rev. Lett. **29**, 1698 (1972).

[21] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).

[22] M. C. Gonzalez-Garcia and Y. Nir, Phys. Lett. **232**, 383 (1989).

[23] F. Pisano and S. Shelly Sharma, Phys. Rev D **57**, 5670

(1998).

[24] J. C. Montero, C. A. de Sousa Pires and V. Pleitez, Phys. Lett. **B502**, 167 (2001); hep-ph/0011296.

[25] E. Ma, Phys. Rev. Lett. **81**, 1171 (1998).

[26] J. Schechter and J. W. F. Valle, Phys. Rev. D **25**, 774 (1982).

[27] M. A. Diaz, M. A. Garcia-Jareno, D. A. Restrepo and J. F. W. Valle, Nucl. Phys. **B527**, 44 (1998).

[28] D. A. Dicus, E. W. Kolb, V. L. Teplitz and R Wagoner, Phys. Rev. D **18**, 1829 (1978). R. N. Mohapatra and P. B. Pal, *Massive Neutrinos in Physics and Astrophysics*, World Scientific, Singapore, 1991; and references therein.

[29] H. Georgi, S. L. Glashow and S. Nussinov, Nucl. Phys. **B193**, 297 (1981).

[30] J. Bockholt *et al.*, Phys. Rev. D **54**, 3641 (1996).

[31] H. Päs, M. Hirsch, H. V. Klapdor-Kleingrothaus and S. G. Kovalenko, Phys. Lett. **B453**, 194 (1999).

[32] R. N. Mohapatra and E. Takasugi, Phys. Lett. **211B**, 192 (1988).

[33] P. Bamert, C. Burguess and R. N. Mohapatra, Nucl. Phys. **B449**, 25 (1995).

[34] W. Rodejohann, Phys. Rev. D**62**, 013011 (2000); H. V. Klapdor-Kliengrothaus, H. Päs and A. Yu. Smirnov, Phys. Rev. D **63**, 073005 (2001).

[35] M. Czakon, J. Gluza and M. Zralek, hep-ph/0003161.

[36] J. C. Montero, V. Pleitez and M. C. Rodriguez, Phys. Rev. D **58**, 094026 (1998); Phys. Rev. D **58**, 097505 (1998); and Int. J. Mod. Phys. **A16**, 1147 (2001).